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On effective actions for the bosonic tachyon

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Abstract: We extend the analysis of [24] to the bosonic case and find the one-derivative effective action valid in the vicinity of rolling tachyons with an energy not larger than that of the original D-brane. For on-shell tachyons rolling down the well-behaved side of the potential in this theory, the energy is conserved and the pressure eventually decreases exponentially. For tachyons rolling down the "wrong" side, the pressure instead blows up in a finite time.

Keywords: Bosonic Strings, Tachyon Condensation.

Contents

1. Introduction

From an analysis [1, 2] using boundary CFT (Conformal Field Theory) techniques [3, 4, 5, 6] it has been possible to deduce different properties of the open-string tachyon.[1] It was subsequently attempted to reproduce these properties from an effective field theory. The "standard form" lagrangian[2]

$$L = -\frac{1}{\cosh\left(\alpha\tilde{T}/2\right)}\sqrt{1+\partial_\mu\tilde{T}\partial^\mu\tilde{T}} \tag{1.1}$$

was proposed [1, 2] and [19]–[23], based on its ability to reproduce some of the properties of the tachyon known from the boundary CFT analysis. Here, $\alpha = 1$ describes the bosonic tachyon, while $\alpha = \sqrt{2}$ corresponds to the superstring case.

Recently, Kutasov and Niarchos [24] managed to derive this action in the superstring case with an ansatz of the type $L(T, \partial_\mu T\partial^\mu T)$ using only two requirements: First, that the action should allow the known tachyon solutions. And second, that it should be consistent with certain results from boundary SFT (String Field Theory).[3]

The purpose of this paper is to try to extend the derivation to the bosonic case. We give the details of this derivation in section 2. In section 3 we investigate what the properties of the resulting action are, and finally in section 4 the validity region of the action is discussed.

[1] Recent progress on both open-string and closed-string tachyon dynamics include [7]–[18].

[2] In this paper, the flat metric is $\eta_{\mu\nu} = (-1, +1, \ldots, +1)$ and $\alpha' = 1$.

[3] A similar argument appeared already in [25].

2. Deriving an effective action for the tachyon

In this section, we will derive the effective action. In section 2.1 we will explain the conditions which are used to determine the lagrangian. In section 2.2 we will then use these conditions to derive the action. The derivation is analogous to that of [24] for the superstring case.

2.1 Conditions

Let us assume that we have a lagrangian of the form $L = L(T, \partial_\mu T \partial^\mu T)$.

We begin by discussing the EOM condition arising from requiring that the equations of motion be satisfied by the rolling tachyon solution, using the formalism of [26]. We write

$$L = L_{\text{even}} + L_{\text{odd}} \tag{2.1}$$

where the even part is

$$L_{\text{even}} = \sum_{n=0}^{n=\infty} L_{2n} \tag{2.2}$$

and the odd part is

$$L_{\text{odd}} = \sum_{n=0}^{n=\infty} L_{2n+1} \,. \tag{2.3}$$

It will turn out that $n = 0$ is sufficient to satisfy the BSFT condition (2.10).

We now introduce the parameter γ to distinguish between the even ($\gamma = 0$) and odd ($\gamma = 1$) cases. Each term in the sums (2.2) and (2.3) goes as $T^{2n+\gamma}$:

$$L_{2n+\gamma} = \sum_{l=0}^{l=l_0} a_l^{(n)} (\partial_\mu T \partial^\mu T)^l T^{2n+\gamma-2l} \,. \tag{2.4}$$

It is known that string theory allows the "S-brane" solution [1, 2]

$$T(t) = T_+ e^{t/\alpha} + T_- e^{-t/\alpha} \tag{2.5}$$

for the tachyon field T in the spatially homogeneous case. We have here introduced another parameter, $\beta \equiv \alpha^2$, which distinguishes between the bosonic ($\beta = 1$) and the superstring ($\beta = 2$) case.

The equations of motion which follow from (2.1), (2.2) and (2.3) are:

$$\sum_{l=1}^{l=l_0} a_l^{(n)} 2l \partial^\mu \left[(\partial_\lambda T \partial^\lambda T)^{l-1} (\partial_\mu T) T^{2n+\gamma-2l} \right] = \sum_{l=0}^{l=l_0} a_l^{(n)} (2n+\gamma-2l)(\partial_\mu T \partial^\mu T)^l T^{2n+\gamma-2l-1} \,. \tag{2.6}$$

As indicated, these should be satisfied by the solutions (2.5) for each n, as the solutions are exponentials. This renders the following recursion relation:

$$a_{l+1}^{(n)} = \frac{\beta}{2} \frac{(2l-1)(2n+\gamma-2l)}{(2l+1)(l+1)} a_l^{(n)} \,. \tag{2.7}$$

We see from this that for the even case, $\gamma = 0$, we get non-zero coefficients only for $l = 0, \ldots, n$, i.e. the upper summation index in (2.4) is $l_0 = n$. For the odd terms, $\gamma = 1$, we get non-zero coefficients for $l = 0, \ldots, \infty$, i.e. $l_0 = \infty$. Note that the lower summation index is always 0, as already indicated in (2.4).

The solution of the recursion relation is given by

$$a_l^{(n)} = \left(\frac{\beta}{2}\right)^l \frac{1}{(2l-1)l!} \frac{(2n+\gamma)!!}{(2n+\gamma-2l)!!} a_0^{(n)} . \tag{2.8}$$

In order to find the indicial coefficients $a_0^{(n)}$, it will be necessary to impose the boundary SFT (BSFT) condition, to which we now turn.

Using BSFT [27]–[30], different tachyon profiles were used in [31]–[33] to extract information about the tachyon lagrangian, and rolling tachyons were studied within this framework e.g. in [34, 35].

According to BSFT [27]–[30] and [36, 37], the on-shell spacetime action $S_{\text{on-shell}}$ is, both in the bosonic and the superstring [32] case, proportional to the disc partition function Z:

$$S_{\text{on-shell}} = cZ\,, \tag{2.9}$$

where c is a constant. The relation we will have use for is instead

$$L_{\text{on-shell}} = cZ'(t)\,, \tag{2.10}$$

where $Z'(t)$ denotes the world-sheet disc partition function with the zero mode t unintegrated. It is consistent in the sense that (2.10) becomes (2.9) upon integrating out the t dependence. We will take $c = -1$, which is required for consistency with the results of [25] (see also [24, 38]).

We will proceed by imposing this condition for the "half S-brane" case $T_- = 0$, i.e. $T = T_+ e^{t/\alpha}$. $Z'(t)$ is then given by [38]

$$Z' = \frac{1}{1 + \frac{1}{\beta}(2\pi)^\beta T^\beta} = \sum_{m=0}^{m=\infty} (-1)^m \frac{(2\pi)^{\beta m}}{\beta^m} T^{\beta m} . \tag{2.11}$$

The terms (2.4) in the on-shell lagrangian, using (2.5), are

$$L_{2n+\gamma}^{\text{on-shell}} = T^{2n+\gamma} \sum_{l=0}^{l=l_0} a_l^{(n)} \frac{(-1)^l}{\beta^l} . \tag{2.12}$$

Imposing (2.10) with $c = -1$, we see that relating equal powers of T requires

$$m = \frac{1}{\beta}(2n+\gamma) . \tag{2.13}$$

Apparently, if $\beta = 2$ (superstring case) then $\gamma = 0$, i.e. only even terms are required. This means that L_{super} will be invariant under $T \to -T$, as can be expected. If $\beta = 1$ (bosonic case) no such restriction appears, i.e. we will need both even and odd terms.

Matching terms in (2.10) corresponding to equal powers of T gives the relation

$$\frac{(-1)^{m+1}}{\beta^m} = \sum_{l=0}^{l=l_0} a_l^{(n)} \frac{(-1)^l}{\beta^l}\,, \tag{2.14}$$

where we have made the field redefinition $T \to 2\pi T$ to get rid of the factors of 2π.

Since $a_l^{(n)}$ is proportional to $a_0^{(n)}$ according to (2.8), this relation uniquely determines the indicial coefficient $a_0^{(n)}$, and that value can then be substituted back into (2.8) to give the unique solution for the expansion coefficents $a_l^{(n)}$.

2.2 Derivation

Let us begin by treating the bosonic even case, which according to section 2.1 corresponds to the parameter values $\beta = 1$, $\gamma = 0$, $l_0 = n$. Performing the sum in (2.14) using (2.8) and (2.13) determines the indicial coefficent:

$$a_0^{(n)} = \frac{(2n-1)!!}{n!2^n}\,. \tag{2.15}$$

Substituting this back into (2.8) gives

$$a_l^{(n)} = \frac{(2n-1)!!}{2^n(2l-1)l!(n-l)!}\,, \tag{2.16}$$

which can be inserted into equations (2.1), (2.2) and (2.4) to give

$$L^{\text{even}}_{\text{bosonic}} = \sum_{n=0}^{n=\infty} \frac{(-1)^n(2n-1)!!}{2^n} \sum_{l=0}^{l=n} \frac{1}{(2l-1)(n-l)!l!} a^{n-l} b^l\,, \tag{2.17}$$

where we have defined $a \equiv -T^2$ and $b \equiv -\partial_\mu T \partial^\mu T$. The lemma (A.1) tells us that this is equal to

$$L^{\text{even}}_{\text{bosonic}} = -\frac{1}{1-T^2}\sqrt{1 - T^2 - \partial_\mu T \partial^\mu T}\,. \tag{2.18}$$

We now continue with the bosonic odd case, which corresponds to $\beta = 1$, $\gamma = 1$, $l_0 = \infty$. Performing the sum in (2.14) using (2.8) and (2.13) determines the indicial coefficent:

$$a_0^{(n)} = -\frac{1}{\sqrt{\pi}} \frac{n!}{(n+1/2)!}\,. \tag{2.19}$$

Substituting this back into (2.8) gives

$$a_l^{(n)} = -\frac{1}{\sqrt{\pi}} \frac{n!}{(n+\frac{1}{2}-l)!} \frac{1}{(2l-1)l!}\,. \tag{2.20}$$

Plugging this result into equations (2.1), (2.3) and (2.4) we get the bosonic odd lagrangian,

$$L^{\text{odd}}_{\text{bosonic}} = -\frac{1}{\sqrt{\pi}} \sum_{n=0}^{n=\infty} n! \sum_{l=0}^{l=\infty} \frac{1}{(n+\frac{1}{2}-l)!(2l-1)l!} a^{2n+1} b^l\,, \tag{2.21}$$

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where we have defined $a \equiv T$ and $b \equiv \frac{\partial_\mu T \partial^\mu T}{T^2}$. We contend that this equals (assuming $T = T(t)$)

$$L^{\mathrm{odd}}_{\mathrm{bosonic}} = \frac{2}{\pi}\frac{1}{1-T^2}\left[\sqrt{1-T^2+\dot{T}^2}\sin^{-1}\left(T\sqrt{1-\frac{\dot{T}^2}{T^2}}\right)+\dot{T}\sin^{-1}\left(\frac{\dot{T}}{T}\right)\right]. \tag{2.22}$$

We believe that (2.21) and (2.22) are identical for the following reason: According to section 2.1, imposing the requirements that the equations of motion should be satisfied by (2.5) and that (2.10) should hold uniquely determines the lagrangian. Consequently, since we know that (2.21) fulfills these conditions, and it can be checked that (2.22) also satisfies these conditions, they must be identical. As a check, we have confirmed that the Taylor expansion of (2.22) in a and b coincides with (2.21).

To get the complete bosonic lagrangian, we need to add the even (2.18) and odd (2.22) parts:

$$\begin{aligned} L_{\mathrm{bosonic}} &= L^{\mathrm{even}}_{\mathrm{bosonic}} + L^{\mathrm{odd}}_{\mathrm{bosonic}} \\ &= -\frac{2}{\pi}\frac{1}{1-T^2}\left[\sqrt{1-T^2+\dot{T}^2}\cdot\cos^{-1}\left(T\sqrt{1-\frac{\dot{T}^2}{T^2}}\right)-\dot{T}\sin^{-1}\left(\frac{\dot{T}}{T}\right)\right] \\ &= +\frac{2}{\pi}\frac{1}{1-T^2}\left[\sqrt{T^2-\dot{T}^2-1}\cdot\cosh^{-1}\left(T\sqrt{1-\frac{\dot{T}^2}{T^2}}\right)+\dot{T}\sin^{-1}\left(\frac{\dot{T}}{T}\right)\right]. \end{aligned} \tag{2.23}$$

The first form is best suited for the field values $0 \le T^2 - \dot{T}^2 \le 1$, while the second is more appropriate when $T^2 - \dot{T}^2 \ge 1$ and positive T (hence, $T \ge 1$). The forms are related by the relation $\cos^{-1}(1+\alpha^2) = i\cosh^{-1}(1+\alpha^2)$. The forms for the bosonic lagrangian given here assume that T only depends on t, but since $\dot{T}$ always appears squared in terms of the Taylor expansion, covariance is manifest.

Note that if we have a half S-brane, i.e. if either T_+ or T_- in (2.5) vanishes, the lagrangian (2.23) reduces on-shell to

$$L_{\mathrm{bosonic}}(\dot{T} = \pm T) = -\frac{1}{1+T} \tag{2.24}$$

by virtue of (2.10). For large $|T|$, T and $\dot{T}$ are exponentially close modulo a sign,

$$\frac{\dot{T}}{T} = \pm 1 + O(e^{-2t}), \tag{2.25}$$

using (2.5). Consequently, in the limit $|T| \to \infty$ the lagrangian (2.23) behaves on-shell as

$$L_{\mathrm{bosonic}} \to -\frac{1}{T}. \tag{2.26}$$

This coincides with the large $|T|$ behaviour of (2.24), as expected since $|\dot{T}| = |T|$ holds exactly there.

To get (2.23) we required that (2.5) be a solution of the equations of motion. Let us mention that if we had instead imposed $\tilde{T} = \sqrt{T_+}e^{t/2} + \sqrt{T_-}e^{-t/2}$ as a solution for $\tilde{T}$ where $\tilde{T}^2 \equiv +T$ (valid for $|T| \gg 2\sqrt{|T_+T_-|}$), the resulting total bosonic lagrangian in T, upon using the lemma (A.1), would have been

$$\tilde{L}_{\text{bosonic}} = -\frac{1}{1+T}\sqrt{1+T+\frac{\partial_\mu T \partial^\mu T}{T}}\,. \tag{2.27}$$

This lagrangian, which contains both even and odd parts, also has the properties (2.24) and (2.26). Its equations of motion in T are satisfied by the half S-brane but *not* by the full S-brane solution (2.5), due to the large T approximation. Making the field redefinition

$$T \equiv \sinh^2\left(\frac{\tilde{T}}{2}\right) \tag{2.28}$$

takes (2.27) to the even standard form (1.1) for $T \geq 0$.

Finally, let us also mention that the superstring case (non-BPS brane in type II string theory), corresponding to $\beta = 2$, $\gamma = 0$, $l_0 = n$, can also be treated in an analogous way, and again using the lemma (A.1), the result is [24]

$$L_{super} = -\frac{1}{1+\frac{1}{2}T^2}\sqrt{1+\frac{1}{2}T^2+\partial_\mu T \partial^\mu T}\,. \tag{2.29}$$

The field redefinition

$$T \equiv \sqrt{2}\sinh\left(\frac{\tilde{T}}{\sqrt{2}}\right) \tag{2.30}$$

then transforms the lagrangian (2.29) into the standard form (1.1).

3. Properties of the bosonic lagrangian

In the superstring case, the lagrangian (1.1) has proven to satisfy several different consistency checks, so in this section we will investigate some of the properties of (2.23); the energy-momentum tensor in section 3.1, and the potential, closed string vacuum and lumps in section 3.2.

3.1 Energy-momentum tensor

We will now compare the behaviour of the energy-momentum tensor of the theory defined by (2.23) to that of the BCFT analysis [1, 2].

We begin by discussing the full S-brane case $\rho < \tau_p$ for which the BCFT energy-momentum tensor is given by [1, 2]

$$\begin{aligned} \rho_{BCFT} &= \tau_p \cos^2(\pi\tilde{\lambda}) \\ (p_{BCFT})_i(t) &= -\tau_p\left[\frac{1}{1+\sin(\pi\tilde{\lambda})e^t}+\frac{1}{1+\sin(\pi\tilde{\lambda})e^{-t}}-1\right], \end{aligned} \tag{3.1}$$

where $i = 1, \ldots, p$. The constant $\tilde{\lambda}$ parameterizes the solutions, and should satisfy[4] $\tilde{\lambda} = \hat{T}_0 + O(\hat{T}_0^2)$, i.e.

$$\tilde{\lambda} = \frac{1}{2\pi}T_0 + O(T_0^2)\,, \tag{3.2}$$

[4] Recall that we rescaled T in equation (2.14). It is really $\hat{T} = \frac{1}{2\pi}T$ which satisfies the BSFT condition, hence $\hat{T}_0 = \frac{1}{2\pi}T_0$.

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where T_0 is the turning point of the tachyon, where it has potential energy only: (2.5) becomes

$$T = T_0 \cosh(t) , \tag{3.3}$$

which is related by a time translation to the solution (2.5) with $T_0^2 = +4T_+T_-$, because for this case, $T^2 - \dot{T}^2 = 4T_+T_- > 0$.

The energy-momentum tensor is defined and calculated as

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-\eta}} \frac{\delta S}{\delta \eta^{\mu\nu}} , \tag{3.4}$$

with our lagrangian given by (2.23). The on-shell energy (3.4) is then

$$\rho \equiv T_{tt} = V(T_0) , \tag{3.5}$$

with the potential $V(T_0)$ given in (3.16). In (3.1), the energy is invariant under $\tilde{\lambda} \to -\tilde{\lambda}$, but (3.5) is odd for small T_0; physical tachyon energies have the wrong sign for $T_0 < 0$. For $T_0 \geq 0$, (3.5) satisfies $0 \leq \rho \leq 1 = \tau_p$, (using (3.17)), as expected from (3.1), and at $T_0 = \tilde{\lambda} = 0$ we get $\rho = \rho_{BCFT} = +1$.

Turning to the pressure, we get from (3.4) that

$$p_i(t) \equiv T_{ii} = L_{\text{bosonic}} , \tag{3.6}$$

where $i = 1, \ldots, p$ and L_{bosonic} is given in (2.23). The numerator N of $p_i(t) = L_{\text{bosonic}}$ in (2.23) on-shell is

$$N = f(T_0) - T f\left(\frac{T_0}{T}\right) . \tag{3.7}$$

The function f is defined by

$$f(x) \equiv \sqrt{1 - x^2} \cos^{-1}(x) \equiv \sum_{n=0}^{n=\infty} a_n x^n , \tag{3.8}$$

where we also defined the Taylor expansion coefficients a_n for $f(x)$. The numerator N vanishes for $T = 1$, but not for $T = -1$, so the numerator may contain a factor of $(T - 1)$. More explicitly, inserting the indicated Taylor expansion of $f(x)$ gives

$$N = (T - 1) \sum_{n=0}^{n=\infty} a_n \frac{T_0^n}{T^{n-1}} G_{n-1}(T) , \tag{3.9}$$

where $G_n(x) \equiv \frac{x^n - 1}{x - 1} = 1 + x + x^2 + \cdots + x^{n-1}$ denotes the geometric sum with n terms.

Consider now the well-behaved side $T \geq 0$. The pressure $p_i(t)$ is regular at $T = +1$, since the numerator contains a factor of $(T - 1)$ (see (3.9)). As the tachyon rolls towards the closed string vacuum $T \to \infty$ we get, using (2.26) in (3.6), that

$$p_i(t) = -\frac{1}{T} \longrightarrow 0 . \tag{3.10}$$

Thus, the pressure vanishes as it should according to (3.1), using (3.17) and making the identification $\pi\tilde{\lambda} = \frac{T_0}{2} + O(T_0^2)$ for sufficiently small T_0, which matches the expectation (3.2).

Consider next the singular side $T \leq 0$. As expected from (3.1), as the tachyon rolls down towards the singularity at $T = -1$, the pressure blows up in a finite time: The discussion after equation (3.8) tells us that the numerator (3.7) of L_{bosonic} in (2.23) does not vanish as $T \to -1$, so $p_i(t) = L_{\text{bosonic}}$ has a simple pole there.

Now, consider what happens around $T = 0$. For small T_0, equation (3.7) becomes

$$N = (1 - T)\left(\frac{\pi}{2} - T_0\right) + O(T_0^2)\,. \tag{3.11}$$

Hence, the pressure is

$$\begin{aligned} p_i(t) &= -\left[1 - \frac{2}{\pi}T_0\right]\frac{1}{1+T} + O(T_0^2) \\ &= -\tau_p\left(1 - \frac{2}{\pi}T_0\right)\left[\frac{1}{1+\frac{T_0}{2}e^t} + \frac{1}{1+\frac{T_0}{2}e^{-t}} - 1\right] + O(T_0^2) \end{aligned} \tag{3.12}$$

using $\tau_p = +1$ from (3.17). We see that the t dependence remains the same as in (3.1) to order $O(T_0^2)$, provided that we replace τ_p by $\tau_p\left(1 - \frac{2}{\pi}T_0\right)$, and make the identification $\pi\tilde{\lambda} = \frac{T_0}{2} + O(T_0^2)$, again in agreement with (3.2).

We will make one final comment before leaving the case $\rho < \tau_p$: Placing the bosonic tachyon at rest at the closed string vacuum, $\tilde{\lambda} = 1/2$, gives vanishing pressure and energy (3.1), and the same is true for (3.5) and (3.10), because the initial conditions $T = \infty$, $\dot{T} = 0$ in (2.5) corresponds to $T_+ = T_- = \infty$, i.e. $T_0 = \infty$.

We now turn to the case $\rho > \tau_p$. For $|\sinh(\pi\tilde{\lambda})| < 1$, the BCFT energy-momentum tensor is given by [1, 2]

$$\begin{aligned} \rho_{BCFT} &= \tau_p \cosh^2(\pi\tilde{\lambda}) \\ (p_{BCFT})_i(t) &= -\tau_p\left[\frac{1}{1+\sinh(\pi\tilde{\lambda})e^t} + \frac{1}{1-\sinh(\pi\tilde{\lambda})e^{-t}} - 1\right], \end{aligned} \tag{3.13}$$

where $i = 1, \ldots, p$ and $\tilde{\lambda}$ parameterizes the solutions. Now, $T^2 - \dot{T}^2 = 4T_+T_- < 0$ on-shell using the solution (2.5), which can be time translated to

$$T = T_0 \sinh(t)\,, \tag{3.14}$$

where $T_0^2 = -4T_+T_-$; the tachyon starts at $T = 0$ with a velocity $\partial_t T = T_0$. As will be discussed in section 3.2, the kinetic term blows up at $T = 0$, so the energy-momentum tensor cannot be well-defined; in general, it has imaginary parts (cf. section 4). Nevertheless, a similar analysis as the one for the case $\rho < \tau_p$ results in a conserved energy and a pressure which blows up at $T = -1$ and decreases as it should (3.13) for large T.

Let us now complete the discussion of the energy-momentum tensor by analysing the half S-brane $\rho = \tau_p$, corresponding to starting the tachyon off at rest displaced slightly from the top of the potential hill. The BCFT energy-momentum tensor for this case is[5] [10, 11, 25, 38]

$$\begin{aligned} \rho &= \tau_p \\ p_i(t) &= -\tau_p \frac{1}{1+\hat{\lambda}e^t}\,, \end{aligned} \tag{3.15}$$

where $\hat{\lambda}$ is not really a parameter in this case; it can be set to unity by time translation. The energy-momentum tensor of our theory is precisely equal to the BCFT one, using $\tau_p = 1$ from (3.17) and making the identification $\hat{\lambda} = T_+$: The energy can be checked by setting $T_0 = 0$ in (3.5). The pressure is also correct, as can be seen by combining (3.6) with (2.24). This is not surprising since (2.24) is a direct consequence of having imposed the condition (2.10). The BCFT properties that the pressure is singular at $T = -1$ and falls off exponentially for $T \to \infty$ therefore carry over automatically to our theory in this case.

3.2 Other properties

Let us begin by examining the potential, which follows immediately from (2.23):

$$V(T) = \frac{2}{\pi}\frac{1}{\sqrt{1-T^2}}\cos^{-1}(T) = \frac{2}{\pi}\frac{1}{\sqrt{T^2-1}}\cosh^{-1}(T)\,, \tag{3.16}$$

where the first form is best for $|T| < 1$, and the second is better for $T > 1$. The potential is shown in figure 1. Note that at $T = -1$ the potential explodes to $+\infty$, and that $V(T \to \infty) \longrightarrow 0$. The energy density of the D-brane at the open-string vacuum can be defined and computed as

$$\tau_p \equiv V(0) = 1\,. \tag{3.17}$$

The odd contribution makes the potential monotonically decreasing. In particular, at $T = 0$, it is proportional to $-T + O(T^2)$ (modulo a constant vacuum energy due to the D-brane) instead of the more traditional $-T^2 + O(T^2)$ for a maximum. To still allow for the solutions (2.5), the kinetic term blows up[6] and changes sign at $T = 0$, making the initial-value problem for rolling tachyons with $\rho > \tau_p$ not well-defined. There cannot therefore be a field redefinition which takes (2.23) to the canonical form even locally; if there were, (2.5) would not have been restricted to $T_+T_- \geq 0$. Still, the solution (2.5) tells us that (2.23) describes a tachyon of mass $m^2 = -1$.

In view of the potential (3.16) and the properties of the energy-momentum tensor discussed in section 3.1, it appears that the following regions can be identified:

- $T = -1$ is the singularity where the pressure blows up.
- $T = 0$ is the open-string vacuum (since (2.5) has been imposed as a solution).
- $T = +\infty$ is the closed-string vacuum.

[5]For concreteness, we study the case $T \sim e^t$, but the case $T \sim e^{-t}$ is handled similarly.

[6]In a sense, the fact that the kinetic term blows up at $T = 0$ is the "glue" that makes a tachyon placed at rest at $T = 0$ stay there.

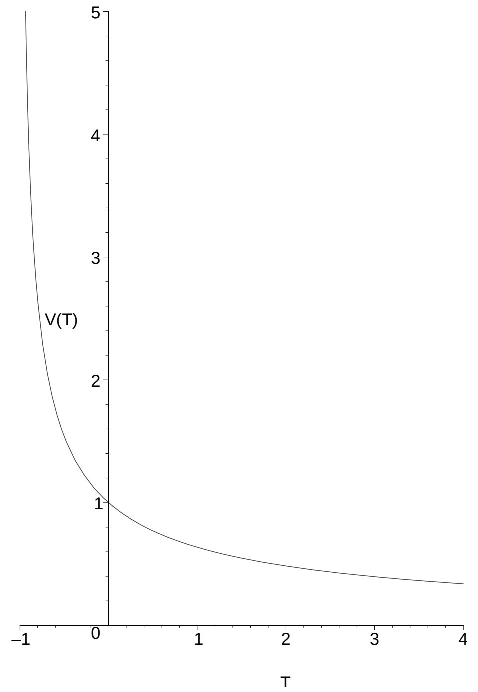

Figure 1: Potential of equation (3.16).

An important expected property of the closed-string vacuum is that it should not admit any open-string particle excitations. As discussed in [40], there are basically two ways in which this property can be realized. In some field theory models [41]–[43] the second derivative of the tachyon potential blows up at the minimum, hence rendering infinite mass to any would-be plane-wave excitations. The second possibility, which we will investigate here analogously to what was done in [40], is that the absence of plane-wave solutions is due to the behaviour of the kinetic term around the minimum (i.e. $T = \infty$), as in [44]–[47].

First, note that in the large T limit, the lagrangian (2.23) becomes

$$-\frac{\pi}{2}L = \frac{1}{T}\left[\ln T\left(1+\frac{1}{2}\frac{\partial_\mu T\partial^\mu T - 1}{T^2}\right) - \frac{\partial_\mu T\partial^\mu T}{T^2}\right] + \cdots,. \qquad (3.18)$$

Making the field redefinitions

$$\begin{aligned} T &\equiv (1+\tilde{T})e^{\tilde{T}} \\ \phi &\equiv e^{-\tilde{T}/2} \end{aligned} \qquad (3.19)$$

turns (3.18) into

$$L = \frac{8}{\pi}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\phi^2\right) + \cdots . \qquad (3.20)$$

In terms of ϕ, which expands around $T = \infty$, the lagrangian has the right canonical form, so this is the field definition to use for analysing excitations around the closed string vacuum. The hamiltonian

$$H = \frac{2}{\pi}\frac{1}{\sqrt{T^2+\partial_\mu T\partial^\mu T - 1}}\cosh^{-1}\left[T\sqrt{1+\frac{\partial_\mu T\partial^\mu T}{T^2}}\right], \qquad (3.21)$$

corresponding to the lagrangian (2.23), should be constant for solutions of the equations of motion, which means that

$$T^2 + \partial_\mu T \partial^\mu T = C \tag{3.22}$$

for some constant C. However, the plane-wave ansatz

$$\phi = ae^{ik\cdot x} \tag{3.23}$$

makes the left-hand side of (3.22) become, using (3.19),

$$\frac{1}{\phi^4}\left[(1-2\ln\phi)^2 - 16k^2(1-\ln\phi)^2\right], \tag{3.24}$$

which is not constant for any $k^2 = -m^2$. We are assuming a flat closed string background, so the concept of particle excitations in itself is well-defined. We therefore conclude that open-string particle excitations are absent at the closed-string vacuum $T = \infty$, as promised.

Finally, let us discuss the viability of lump solutions. In the superstring case, the Dirac-Born-Infeld action has been shown to allow kink solutions [39] (stable D-brane configurations). In the bosonic case, there should instead exist lump solutions (unstable D-branes). However, the lagrangian (2.23) is designed to be valid for profiles in the vicinity of rolling tachyons (cf. section 4), and we can actually show that it does not contain lumps of the type locally resembling the kink solutions of [39].

By analogy with the superstring case [39], let us begin by analysing the energy-momentum tensor, which in the bosonic time-independent case $T = T(x)$ becomes

$$\begin{aligned}
T_{\alpha\beta} &= \eta_{\alpha\beta} L_{\text{bosonic}} \\
\rho \equiv T_{tt} &= -L_{\text{bosonic}} \\
p \equiv T_{xx} &= -\frac{2}{\pi}\frac{1}{\sqrt{1-T^2-T'^2}}\cos^{-1}\left(T\sqrt{1+\frac{T'^2}{T^2}}\right) \\
&= -\frac{2}{\pi}\frac{1}{\sqrt{T^2+T'^2-1}}\cosh^{-1}\left(T\sqrt{1+\frac{T'^2}{T^2}}\right),
\end{aligned} \tag{3.25}$$

where $\alpha,\beta = 0,\ldots,p-1$, $x^p \equiv x$. The pressure p is given in two forms, depending on whether $T^2 + T'^2 \leq 1$ or $T^2 + T'^2 \geq 1$, respectively. Since we are looking for time-independent lump solutions, $T = T(x)$, we should substitute $\dot{T} \to \pm iT'$ in the lagrangian (2.23). The conservation law $\partial^\alpha T_{\alpha\beta} = 0$ now reduces to

$$\frac{\partial p}{\partial x} = 0\,. \tag{3.26}$$

Using (3.25), this means that to have a solution we need to have either[7] $T = A\cos(x) + B\sin(x)$ (which is the Lorentz rotated version of (2.5)), or

$$T^2 + T'^2 \to \infty\,. \tag{3.27}$$

[7] This solution may also correspond to D-branes in the world-sheet theory [48], but we do not analyse it further here.

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As in [39], it is clear that for example a linear profile of infinite slope would locally satisfy (3.27) for $T > -1$. However, we can never find a profile that satisfies (3.27) globally, since a lump coming in from $T = +\infty$ must eventually turn at some finite T. Analysing lumps therefore seems to require higher derivative corrections to the lagrangian (2.23).

4. Discussion

Our main result is the effective lagrangian (2.23) for the bosonic open-string tachyon. It is the unique covariant lagrangian not containing second or higher order derivatives whose equations of motion are satisfied by the solutions (2.5) and which on-shell is proportional to the disc partition function (with zero modes unintegrated) on half S-branes.

In the superstring case (non-BPS brane in type II string theory), this procedure leads to (2.29) which is related by a field redefinition to the standard form (1.1). According to [24], it describes tachyon profiles of the type

$$T = T_+(x^\mu)e^{t/\alpha} + T_-(x^\mu)e^{-t/\alpha} \tag{4.1}$$

(with $\alpha = \sqrt{2}$) where $|\partial_\mu T_+| \ll 1$ and $|\partial_\mu T_-| \ll 1$, expanded around a solution with $T_- = 0$ (or $T_+ = 0$). In particular, it has one free parameter $a_0^{(n)}$ at each order T^{2n} (see (2.8)), precisely as the $2n$-point function of tachyons at order momentum squared in a spatially homogeneous background of half S-branes (i.e. $T_- = 0$). The bosonic lagrangian (2.23) should then analogously be designed to describe the same type of profiles (4.1) (with $\alpha = 1$) for all values of $T > -1$, provided that $|\dot{T}| \leq |T|$. We will explain these conditions shortly.

We also noted in section 2.2 that the bosonic lagrangian (2.27) could be derived by a similar procedure valid for $|T| \gg 2\sqrt{|T_+T_-|}$ (see the discussion around equation (2.27)), and it therefore describes profiles of the type (4.1) when $|T| \gg 2\sqrt{|T_+(x^\mu)T_-(x^\mu)|}$. Since it was related to the standard form (1.1) by the field redefinition (2.28), it appears that (1.1) can only be expected to describe bosonic profiles well in the vicinity of rolling tachyons for large positive T (cf. the discussion in [19]).

The most important test of (2.23) was the behaviour of the physical energy-momentum tensor (section 3.1). The pressure blew up in a finite time for tachyons rolling towards the singular side of the potential, and at the other side the pressure eventually decreased exponentially, as expected. Quantitative agreement was also good for half S-branes or full S-branes with either $T_0 \to 0$ or $T \to \infty$, i.e. close to half S-branes, which of course lies in line with the expectation (4.1).

In general, the lagrangian (2.23), and therefore also the physical energy-momentum tensor (cf. section 3.1), acquires imaginary parts outside of the region $T \geq -1$ and $|\dot{T}| \leq |T|$. The former restriction corresponds to going past the singularity at $T = -1$ where the pressure blows up into the unphysical region $T < -1$. The latter restriction is related to the fact that for rolling tachyons of the type $T = T_0 \sinh(t)$, the kinetic term blows up at $T = 0$, i.e. tachyons which traverse the top of the potential hill cannot be assigned a well-defined energy. The singularity of the kinetic term can be traced back to the fact that the EOM condition in section 2.1 made the lagrangian non-analytic in T. Indeed, the Taylor expansion in $b = -(\dot{T}/T)^2$, given in (2.21), diverges by the quotient test for $|\dot{T}| > |T|$, leading us to restrict the validity region to $|\dot{T}| \leq |T|$. Analytic continuation beyond this region, while possible, introduces imaginary components.[8]

Imposing the BSFT condition (2.10) for full S-branes would rid the lagrangian of imaginary parts. Since the freedom of the ansatz $L(T, \partial_\mu T \partial^\mu T)$ was saturated already by half S-branes, this would require adjusting the ansatz such that more freedom is provided. The right type of freedom could possibly be provided by allowing for higher derivatives. This would also change the equations of motion, possibly in such a way so as to remove the non-analyticity in T.

However, perhaps there is another ansatz than $L = L(T, \partial_\mu T \partial^\mu T)$ which contains the same amount of freedom. Suppose, for example, that we make the ansatz $L = L(T, \partial_\mu T \partial^\mu T, \partial^2 T)$. Instead of (2.4), we then get

$$L_{2n+\gamma} = \sum_{l=l_0}^{l=l_1} \sum_{m=m_0}^{m=m_1} a_{lm}^{(n)} (\partial_\mu T \partial^\mu T)^l (\partial^2 T)^m T^{2n+\gamma-2l-m} \tag{4.2}$$

at order $T^{2n+\gamma}$. Requiring that the equations of motion are consistent as described in section 2.1 leads to

$$\begin{aligned} 0 = {} & a_{l+1,m}^{(n)} 2(m-1)(l+1)(2l+1) + \\ & + a_{l,m}^{(n)} \left[(2n+\gamma-2l)2l(m-1) + m(2n+\gamma-2l-1)(2l+1) + (2n+\gamma-2l-m)\right] + \\ & + a_{l-1,m}^{(n)} m(2n+\gamma-2l+1)(2n+\gamma-2l)\,. \end{aligned} \tag{4.3}$$

The case $m = 0$ is the case already treated in section 2.1, i.e. $a_{l,0}^{(n)} = a_l^{(n)}$ in (2.7) (with only one free parameter, $a_0^{(n)}$). But we see from (4.3) that there is another possibility, $m = 1$, which also has only one free parameter at each order. This example therefore shows that there may be higher derivative lagrangians which also describe profiles of the type (4.1), possibly without the restriction $|\dot{T}| \leq |T|$.

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A. A useful Taylor expansion

We want to show that

$$L \equiv \frac{1}{1+a}\sqrt{1+a+b} = \sum_{n=0}^{n=\infty} \frac{(-1)^{n+1}(2n-1)!!}{2^n} \sum_{l=0}^{l=n} \frac{1}{l!(n-l)!(2l-1)} a^{n-l} b^l\,. \tag{A.1}$$

[8] However, note that static profiles $T = T(x)$ are not restricted to $|T'| \leq |T|$.

This formula was basically derived as a part of [24], but since it is used several times in section 2.2, we repeat the derivation here. Begin by Taylor expanding the left-hand side in a and b and use that $(-1)! = (-2)! = \cdots = \infty$ to drop terms give

$$L = \sum_{n=0}^{n=\infty}(-1)^n \sum_{m=0}^{\infty} \frac{(2m-3)!!(-1)^{m+1}}{2^m} \sum_{k=0}^{k=p} \frac{1}{k!(m-k)!} a^{n+m-k} b^k \,. \tag{A.2}$$

Upon substituting $p \equiv n+m$,

$$L = \sum_{p=0}^{p=\infty}(-1)^{p+1} \sum_{k=0}^{k=p} \frac{1}{k!} \sum_{m=0}^{m=p} \frac{(2m-3)!!}{2^m(m-k)!} a^{p-k} b^k \,. \tag{A.3}$$

The last term can be rewritten using $m \to m+k$ and the identity

$$\sum_{m=0}^{m=p-k} \frac{[2(m+k)-3]!!}{2^m m!} = \frac{2^{k-p}(2p-1)!!}{(p-k)!(2k-1)} \,, \tag{A.4}$$

which collapses one of the sums. This leaves us with

$$L = \sum_{p=0}^{p=\infty} \frac{(-1)^{p+1}(2p-1)!!}{2^p} \sum_{k=0}^{k=p} \frac{1}{k!(p-k)!(2k-1)} a^{p-k} b^k \,, \tag{A.5}$$

which is the same as the right-hand side of (A.1), upon replacing $p \to n$ and $k \to l$.

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